Experimental local realism tests without fair sampling assumption

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Abstract. Following the theoretical suggestion of [E. Santos, Phys. Lett. A **327**, 33 (2004); E. Santos, Eur. Phys. J. D **42**, 501 (2007)], we present experimental results addressed to test restricted families of local realistic models, but without relying on the fair sampling assumption.

1 Introduction

The quest for a conclusive test of local realism has now a pluridecennial history [1], beginning with the celebrated 1965 Bell's paper [2] demonstrating that every Local Hidden Variable Theory (LHVT) cannot reproduce all the results of quantum mechanics when correlation among measurements on entangled states are considered. In particular, some inequalities always satisfied by every LHVT can be violated by Standard Quantum Mechanics (SQM).

Since then many experiments have been realized for testing Bell inequalities [1], with a recent intensification due to the application to the emerging field of quantum information. Nevertheless, whilst a clear spacelike separation has been unequivocally achieved, still a conclusive test is missing, because of low detection efficiency that requires the additional hypothesis that the observed sample is a faithful representation of the whole one (the so-called *fair sampling* assumption, leading to the detection loophole).

In the last years relevant progresses for eliminating this loophole have been realized [1], both based on improvements of detectors and on the result that for non-maximally entangled states the limit for an elimination of this problem is lowered from 82% to 66.7% [3,4]. However, a conclusive experiment is still missing.

On the other hand, specific local realistic models built for surviving traditional Bell inequalities tests, as stochastic optics [5] (a branch of stochastic elecrodynamics [6]), have been falsified by specifically addressed experimental tests [1,7,8].

Recently, some works appeared [9,10] with the purpose to propose some specific inequality verified by restricted families of LHVT, but whose test, realizable by exploiting polarization entangled photon pairs generated by parametric down-conversion (PDC), would be free from the detection loophole. The aim of this paper is to present an experimental test of these theoretical proposals.

2 Theoretical motivations

In [9,10] a set-up based on polarization bipartite entangled states of photons is considered, where one measures the single count rates R_1 and R_2 on the idler and signal arm, the coincidence rate R_{12} and the total pair production rate R_0. One has

$$R_1 = R_2 = \frac{1}{2}\eta R_0 \qquad R_{12} = \frac{1}{4}\eta^2 R_0[1 + V\cos(2\phi)] \quad (1)$$

where ϕ is the angle between the two polarization axes, V the observed visibility and η the detection efficiency (considered identical for the two arms).

The results of such an experiment can be reproduced by a LHV model constituted by the functions [9]:

$$\rho(|\lambda_1 - \lambda_2|) \geq 0 \qquad 0 \leq P_j(|\lambda_j - \phi_j|) \leq 1 \qquad j = 1, 2$$

$$\int \rho(|\lambda_1 - \lambda_2|)d\lambda_1 d\lambda_2 = 1 \qquad (2)$$

where λ_1, λ_2 are the hidden (angular) variables and ϕ_1, ϕ_2 the angles of the polarization planes with respect to an axis perpendicular to pump direction and polarization.

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The predictions for the single count and coincidence rates are:

$$\frac{R_j}{R_0} = \int \rho(|\lambda_1 - \lambda_2|) P_j(|\lambda_j - \phi_j|) d\lambda_1 d\lambda_2 \qquad (3)$$

$$\frac{R_{12}}{R_0} = \int \rho(|\lambda_1 - \lambda_2|) \prod_{j=1}^{2} P_j(|\lambda_j - \phi_j|) d\lambda_1 d\lambda_2. \qquad (4)$$

From this model, introducing the visibility parameters:

$$V_A = \frac{R_{12}(0) - R_{12}(\pi/2)}{R_{12}(0) + R_{12}(\pi/2)}$$

$$V_B = \sqrt{2} \frac{R_{12}(\pi/8) - R_{12}(3\pi/8)}{R_{12}(\pi/8) + R_{12}(3\pi/8)} \qquad (5)$$

one derives the inequality [9]:

$$\frac{V_B}{V_A} \geq F = 1 + \cos^2\left(\frac{\pi\eta}{2}\right)\left[V_B - \frac{\sin^2(\pi\eta/2)}{(\pi\eta/2)^2}\right] \qquad (6)$$

that can be violated in quantum mechanics (where, in the ideal case, $V_B/V_A = 1$). An accurate test of this inequality will be the first achievement of our experiment [11].

Then we consider another inequality of the same type, but valid for a larger family of LHVT [10]. In this case, one takes into account a set of four hidden variables $\lambda \equiv \{\chi_1, \chi_2, \mu_1, \mu_2\}$ (of two different types) instead of two. The equations of this local realistic model are [10]:

$$\rho(\lambda) = \rho_\chi(|\chi_1 - \chi_2|) g_1(\mu_1) g_2(\mu_2) \geq 0$$

$$\int \rho_\chi(|\chi_1 - \chi_2|) d\chi_1 d\chi_2 = \int g_j(\mu_j) d\mu_j = 1$$

$$0 \leq P_j(|\chi_j - \phi_j|) = \int Q_j(\mu_j, |\chi_j - \phi_j|) g_j(\mu_j) d\mu_j \leq 1 \qquad (7)$$

and give the expected ratios (after integrating over the μ_j variables):

$$\frac{R_j}{R_0} = \int \rho_\chi(|\chi_1 - \chi_2|) P_j(|\chi_j - \phi_j|) d\chi_1 d\chi_2 \qquad (8)$$

$$\frac{R_{12}}{R_0} = \int \rho_\chi(|\chi_1 - \chi_2|) \prod_{j=1}^{2} P_j(|\chi_j - \phi_j|) d\chi_1 d\chi_2. \qquad (9)$$

Let us consider a system with rotational invariance, in which the coincidence rates depend only on the relative polarization orientation $\phi = \phi_1 - \phi_2$, and put the probabilities $P_1 = P_2 = P$ (perfect symmetry between the apparatuses on the two branches of PDC). By introducing the parameter (where $()_+$ means putting zero if the quantity inside the bracket is negative):

$$D(\eta) \equiv \frac{4}{3\pi} \sqrt{\frac{2}{3\eta} - \frac{1}{2} - \frac{\sin^4(\pi\eta/2)}{(\pi\eta/2)^4}} \left(V - \frac{\sin^2(\pi\eta/2)}{(\pi\eta/2)^2}\right)_+^{\frac{3}{2}} \qquad (10)$$

with:

$$V = 2 \frac{\sum_{j=1}^{n} R_{12}(\phi_j) \cos(2\phi_j)}{\sum_{j=1}^{n} R_{12}(\phi_j)} \qquad (11)$$
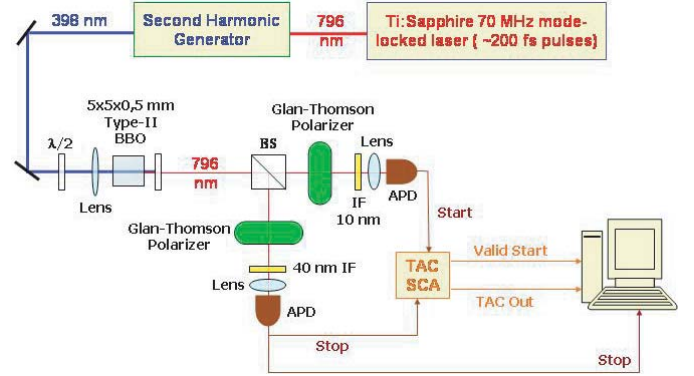

Fig. 1. (Color online) Scheme of the experimental set-up.

one derives the following inequality:

$$\Delta_{min} = \left[\frac{n \sum_{j=1}^{n} R_{12}^2(\phi_j)}{\left(\sum_{j=1}^{n} R_{12}(\phi_j)\right)^2} \right.$$

$$\left. - 2 \frac{\left(\sum_{j=1}^{n} R_{12}(\phi_j) \cos(2\phi_j)\right)^2}{\left(\sum_{j=1}^{n} R_{12}(\phi_j)\right)^2} - 1 \right]^{(1/2)} \geq D(\eta) \qquad (12)$$

where $\phi_j = \pi(j-1)/n$.

Testing this second inequality will be the second purpose of the present work.

3 Experimental set-up and results

In order to test these inequalities, we measured the variation of the coincidence rate $R_{12}(\phi)$ with respect to the variable ϕ, using the experimental set-up reported in Figure 1.

In our set-up a 0.2 W, 398 nm pulsed (with 200 fs pulses) laser beam, obtained by second harmonic generation from a titanium-sapphire beam at 796 nm, pumps a type-II BBO $5 \times 5 \times 0.5$ mm crystal. With a proper selection of the phase-matching conditions, we obtain the collinear degenerate (796 nm) PDC and send it to a non-polarizing 50%–50% beam splitter (BS). Passing through the BS, the state changes like this:

$$|HV\rangle \rightarrow \frac{1}{2}\Big(|H_r V_r\rangle + |H_t V_t\rangle + |V_r H_t\rangle + |H_r V_t\rangle\Big), \quad (13)$$

where the pedices r and t respectively indicate the reflection and transmission directions.

By postselecting the cases when photons follow different paths, one obtains the Bell state:

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}\Big(|V_r H_t\rangle + |H_r V_t\rangle\Big). \qquad (14)$$

On the transmission (t) arm we put a Glan-Thomson polarizer, a pinhole, an interference filter (FWHM 10 nm,
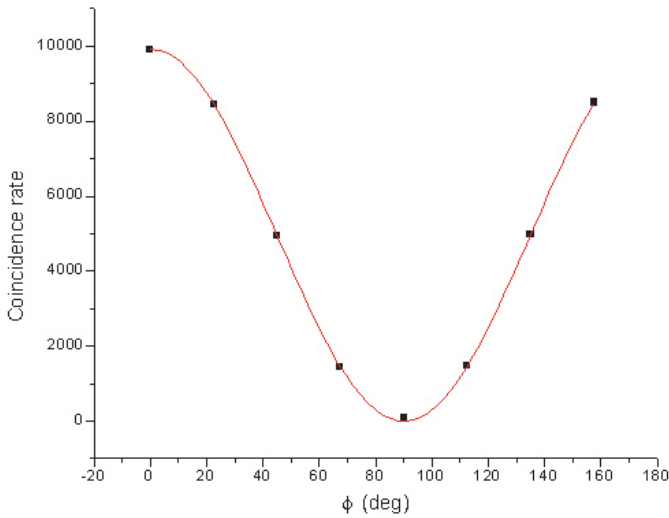
Fig. 2. Coincidence rates versus ϕ (angle between the axes of t and r polarizers). The experimental data are compared with the theoretical curve $\cos^2(\phi)/2$, with whom are in perfect agreement (correlation coefficient $r = 0.9998$). Uncertainty bars are too small for being visible.

λ_{peak} 795 nm) and an optical fiber coupler connected with a multi-mode fiber to the detector, a Perkin-Elmer SPCM-AQR-14-FC-9491 silicon avalanche photo-diode (quantum efficiency at 796 nm: 62%); the reflection (r) arm featured a Glan-Thomson polarizer, a large band interference filter (FWHM 40 nm, λ_{peak} 800 nm) and the same fiber coupling and detection devices as in the other arm. Taking the t photons as trigger, a coincidence window is opened on the arm r detector to reveal the correlated pairs: this is obtained by sending the output of the t detector as start to a Time-to-Amplitude Converter (TAC) that receives the r detector signal as stop. The 20 ns coincidence window is set properly to exclude spurious coincidences with PDC photons of the following pulse (we remind that the repetition rate of the laser is 70 MHz). The TAC output is then acquired via PC, together with the TAC's Valid Start counts (giving us the total number of opened coincidence windows). The background has been evaluated and subtracted by measuring the TAC output of the system without PDC generation (simply rotating the pump polarization by 90° before the nonlinear crystal).

To violate the inequalities presented in the previous section, we fixed the polarization axis of both the Glan-Thomson polarizers at an angle of $\frac{\pi}{4}$ with the horizontal plane, and then we performed a scan of the coincidence rates $R(\phi)$ (where ϕ represents the difference between the polarization planes) simply rotating the polarizer on the r arm by $\frac{\pi}{8}$ per step, until we cover the whole interval $\phi \in [0, \pi[$: the rate obtained for each step is the average of 30 different acquisitions of 30 s each.

The obtained curve is shown in Figure 2.

As a first test for our entangled state, we checked (of course within the fair sampling hypothesis) the

Clauser-Horne inequality [1], written in the form:

$$\frac{\left|R(\frac{\pi}{8}) - R(\frac{3\pi}{8})\right|}{R_{tot}} \leq \frac{1}{4}. \quad (15)$$

The collected data, with a measured visibility 97.8%, led to the following violation:

CH violation	σ	violation/σ
0.1026	0.0021	47.8

Then, to violate the inequality (6), we evaluated the quantities:

V_B	V_A
0.9985 ± 0.0030	0.9784 ± 0.0017
V_B/V_A	F
1.0205 ± 0.0048	1.0876 ± 0.0009

where, following the indications of reference [10], we used for the value of the quantum efficiency the nominal detector efficiency as reported on the manufacturer data sheet for our wave length, $\eta = 62\%$.

Of course, the difference of V_A, V_B from unit and from each other derives from unavoidable imperfections of the experimental apparatus; nevertheless a clear violation of the inequality (6) is observed:

$F - V_B/V_A$	σ	violation/σ
0.0671	0.0057	11.7

Finally, we present the results for the inequality (12) (with again $\eta = 62\%$). Our data led to a violation:

$D(\eta) - \Delta_{min}$	σ	violation/σ
0.0073	0.0022	3.3

For the sake of completeness, we mention that both the inequalities (6, 12) still are violated even without background subtraction (of 11.6 and 1.7 standard deviations respectively).

Thus, we have clearly shown a violation of the predictions made upon the LHVT models presented in [9,10], whilst perfectly agreeing with SQM.

4 Conclusions

In summary, we have experimentally demonstrated, following the theoretical predictions of [9,10], that the specific families of hidden variable theories considered there fail to properly reproduce the observed correlations between measurements of bipartite polarization photon entangled states, and are therefore falsified by our experiment.

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